Qualifying problems for WWW

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Abstract

What is the first law of *thermodynamics*?
You don't talk about thermodynamics.

This series of lectures will break any taboos, exposing participants to the basic concepts of statistical mechanics - the theory that is fundamental to our understanding of Physics. The links to dynamics, quantum mechanics, and magnetism will also be shown. Nevertheless... my love for Physics behoves me to start with a bunch of well-known jokes and check whether we have the same sense of humour. Let's go!

The list of problems is insightful but not exhaustive - be prepared to go beyond its scope.

The course is based on the syllabus of "Statistical Mechanics" by Stanford University.

Marking scheme

- Each question from the *You do you* section is worth 10 points. It is enough to answer them shortly to get a full mark. The remaining problems are worth 4 points each.
- The grading will be as follows:
  - 4 points - The answer is full, and it is bluntly seen that the concept is well understood by the participant.
  - 3 points - There are minor mistakes, but the topic is well understood.
  - 2 points - The answer is partial/mechanical, or all Maths is good but the concept isn't understood.
  - 1 points - It's bad, but I won't be a bad guy$^{\text{TM}_{BillyEilish}}$ - it means that you wrote something meaningful in the end.
  - 0 points - No response, or complete bullshit, or the answer is illegible.
- The answers may be prepared in English or Polish.
- I will create a group-chat on FB Messenger so you can discuss the problems and get to know each other before meeting irl. Send me an e-mail if you want to join. Disclaimer: you can share materials and questions there, but discussing solutions in detail is strictly forbidden and results in disqualification.
- There will be no qualification threshold but I would recommend you to attend only if you scored at least 70 points (desirably, at least 30 from *Physics* and 20 from *Maths*).
- The answers should be sent to *marcelmordarski@gmail.com* in a single PDF file by May, 17^{th}, 23:59:59 Polish time. Although computer-typed answers are preferred, I'm ok with handwritten answers if your handwriting is legible (your teachers must've stated this at least once during high school!). Otherwise, type it on the computer.
- Don't be afraid to write comments! Therein lies the success, actually.
- As long as you submit your own work (I'll check it...), I don't care how you get the knowledge to solve these problems. University approved websites, textbooks, and friends who may help you get the idea (and nothing more!) are recommended.

You do you

1. This is the toughest one: write five sentences about yourself, so I can get a feeling who the hell you are.
2. What's your command of English? Would you prefer lectures to be run in Polish or English?

Physics

1. Explain to a kindergarten child the zeroth, the first, and the second law of thermodynamics. Which one is the most crucial?
2. Describe as strictly as you can the phase space of the molecules in your body. What's its volume?
3. Write the definitions of temperature and pressure from your Physics textbook and show their flaws.
4. Compare ideal gas to weakly interacting one.
5. How do we define a phase transition? Which state function tells us a phase that we observe in particular conditions?
6. Let k be a constant. Derive a formula for the rate of heat exchange of a substance between two concentric cylindrical surfaces in the radial direction. Draw a picture of this situation!
7. Why do we have to add heat to melt ice in a constant temperature?
8. The free mean path of ammonia molecules in the standard conditions is s. There are N molecules per cubic centimeter in this gas. What's the radius of the molecule?
9. Prove logically that - even though they thought about different formulations of the second law - Clausius and Planck outlined the same idea.
10. Convince me that the efficiencies of the Carnot and Otto engines are $\eta = \frac{T_1 - T_2}{T_1}$ and $\eta = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma - 1}$ respectively.

Maths

1. Prove that $\log ab = \log a + \log b$ for $a, b > 0$, and $\log_c d^f = f \log_c d$ for $c \in (0,1) \cup (1, \infty), d > 0, f \in \mathbb{R}$.
2. Prove that functions $f(x) = e^x$ and $h(x) = \frac{1}{\log_{1-x} 392}$ are smooth. Express them as power series around $x = 37$.
3. Read a section on multi-variable calculus in any mathematical textbook, and then, using the method of Lagrange's multipliers, find the dimensions of the box with the largest volume if the total surface area is $64cm^2$.
4. Calculate the differential of the state function from the exercise *Physics 5*. Likewise, evaluate the differential of Helmholtz free energy.
5. Calculate $\int_{-\infty}^{\infty} e^{-x^2} \mathrm{d}x$, $\int \log x \mathrm{d}x$, $\int_{-10^{-4}}^{10^4} \frac{1}{x} \mathrm{d}x$ and $\int_{-1}^{2} x \cdot 1005^{x^2} \mathrm{d}x$ - there are no mistakes that I know about in this problem.
6. Define chaos mathematically. Using your definition, describe the propagation of mess in your bedroom.
7. Define a consistent axiomatic system. Does there exist a bijection $\mathbb{N} \to \mathbb{R}$? Why? Invent your own story resembling *The barber paradox*.
8. Give in your own words the combinatoral argument behind binomial theorem. Add $\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n}$, and prove your result inductively.

9. Prove the multinomial coefficient formula.

10. Show that $\mathrm{d}\sinh x = \cosh x \mathrm{d}x$, using definitions of these functions. Sketch $y = \frac{\tanh\left(7x+\frac{\sqrt{17}}{2}\right)}{\pi} + \sqrt[209]{2020}$, starting from hyperbolic tangent. Show all transformations in one grid.

Recommended materials

- K. F. Riley, M. P. Hobson, S. J. Bence - "Mathematical Methods for Physics and Engineering" - https://luiarthur.github.io/assets/ams211/mathbook.pdf
- R. Resnick, D. Halliday - "Physics"